COMMUNICATIONS TO THE EDITOR

The Effect of Vapor-Flow Gradient on Distillation Plate Efficiency

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The Murphree efficiency of a distillation tray is often higher than the corresponding point efficiency. One cause of the increase is the presence of a concentration gradient across the tray. The effect of concentration gradient for a simple model of a distillation tray has been treated by W. K. Lewis, Jr., (*1*). He showed how Murphree efficiencies of greater than 100% can be encountered under certain operating conditions, but a great number of simplifying assumptions were made to arrive at a mathematically tractable form. These assumptions were:

1. One-dimensional flow
2. Complete vertical mixing in the liquid
3. No axial mixing in the liquid
4. Binary mixture
5. Constant molal overflow
6. Uniform vapor distribution across tray
7. Well-mixed vapor under tray
8. Straight equilibrium line
9. Constant Murphree point efficiency.

Of these nine assumptions the sixth and third would seem to be the most restrictive. In this paper the sixth assumption (uniform vapor distribution) has been eliminated, and the significance of nonuniform vapor flow has been demonstrated. In all cases the nonuniform vapor flow reinforces the effect of the concentration gradient, and the resulting Murphree tray efficiency is higher than would be expected on the basis of Lewis' analysis.

THE GENERAL EQUATION

Integrating the material balance on a differential slice of the tray, and assuming the value of the concentration in the exit liquid is known, one can arrive at the following form for the vapor-phase concentration as a function of the distance across the tray:

$$\ln \frac{y - y_{n-1}}{y_0 - y_{n-1}} = \int_0^1 \frac{mEg}{L} dw - \int_0^w \frac{mEg}{L} dw \qquad (1)$$

If one assumes that mE/L is constant across a tray, and notes that $\int_0^1 g\, dw$ is $\bar{G}$, one arrives at the following form:

$$y = y_{n-1} + (y_0 - y_{n-1}) \exp\left\{ \frac{mE\bar{G}}{L}\left(1 - \int_0^w \frac{g}{\bar{G}} dw\right)\right\} \qquad (2)$$

The Murphree tray efficiency is defined by

$$E_{mV} = \frac{\bar{y} - y_{n-1}}{y_0^* - y_{n-1}} \qquad (3)$$

The average vapor concentration is $\int_0^1 y\, dw$. Thus one finally obtains:

$$\frac{E_{mV}}{E} = \int_0^1 \exp\left\{ \frac{mE\bar{G}}{L}\left(1 - \int_0^w \frac{g}{\bar{G}} dw\right)\right\} dw \qquad (4)$$

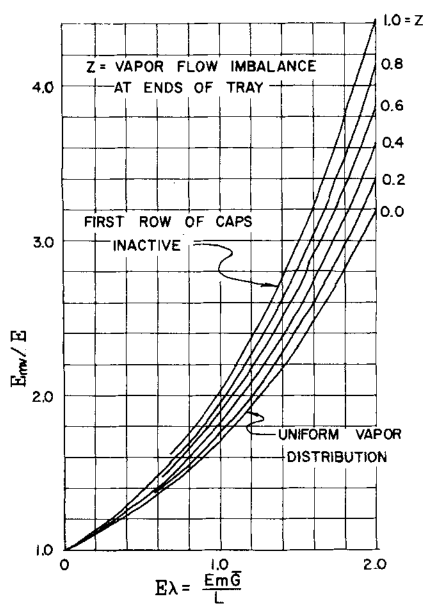

Fig. 1. Combined effect of concentration and vapor-flow gradients on distillation plate efficiency.

This general form is valid for any variation of vapor flow, $g(w)$. For a uniform vapor flow g is $\bar{G}$, and the equation reduces to that treated by Lewis.

LINEAR VAPOR-FLOW GRADIENT

To determine if the effect of vapor-flow gradient is significant the assumption of a linear vapor flow gradient was made:

$$g/\bar{G} = 1 - z(1 - 2w) \qquad (5)$$

Substituting in (4) one obtains

$$\frac{E_{mV}}{E} = \int_0^1 \exp\left\{ - E\lambda z\left[w^2 + w\left(\frac{1-z}{z}\right) - \frac{1}{z}\right]\right\} dw \qquad (6)$$

By completing the square and changing variable one arrives at the following form:

$$\frac{E_{mV}}{E} = \frac{\frac{\sqrt{\pi}}{2} \exp\left\{E\lambda - E\lambda z\left(\frac{1-z}{2}\right)^2\right\}}{\sqrt{E\lambda z}} \left[\operatorname{erf} \sqrt{E\lambda z}\left(\frac{1+z}{2z}\right) - \operatorname{erf} \sqrt{E\lambda z}\left(\frac{1-z}{2z}\right) \right] \qquad (7)$$

The corresponding solution for no vapor-flow gradient is

$$\frac{E_{mV}}{E} = \frac{\exp E\lambda - 1}{E\lambda} \qquad (8)$$

The rather complicated form (7) was programmed for the Bendix G-15 digital computer with DAISY 201 interpretive routines.

RESULTS AND DISCUSSION

The calculated values of efficiency ratio are shown in Figure 1. If one plots the efficiency ratio E_{mV}/E vs. $E\lambda$, all the values given by Lewis for one tray fall on the single line in Figure 1 labelled "uniform vapor distribution."

The following condensed table gives the efficiency ratio for the limiting vapor distributions at various values of $E\lambda$:

$E\lambda$	$(E_{mv}/E)^0$	$(E_{mv}/E)^1$
0.5	1.30	1.37
1.0	1.70	2.00
1.5	2.32	2.96
2.0	3.20	4.42

0—Uniform vapor distribution
1—First row of caps inactive

As in Lewis' analysis the effect is more pronounced at higher values of $E\lambda$, and as would be expected the effect increases monotonically with the amount of imbalance in the vapor flow. The error in the predicted tray efficiency can be as high as 20 or 30% if the vapor flow is sufficiently nonuniform. Figure 1 can be used to determine the efficiency ratio for combined vapor flow and concentration gradients.

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NOTATION

E = Murphree point efficiency
E_{mV} = Murphree tray efficiency
g = point value of the vapor flow rate
$\overline{G}$ = flow rate averaged over entire tray
L = liquid flow rate
m = slope of equilibrium line, $y^* = mx + b$
w = distance along the tray, as a fraction of the total
y = vapor composition
y_{n-1} = vapor composition under the tray
y_0 = vapor composition at exit of tray ($w = 1$)
y^* = vapor composition in equilibrium with liquid at any point
$\overline{y}$ = vapor composition averaged over entire tray
z = fractional imbalance in vapor flow rate at end of tray (for linear vapor-flow gradient) $= \dfrac{\overline{G} - g}{\overline{G}}$ evaluated at $w = 0$
λ = $m\overline{G}/L$

LITERATURE CITED

1. Lewis, W. K. Jr., *Ind. Eng. Chem.*, **28**, 399-402 (1936).

A Note on Transport to Spheres in Stokes Flow

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Mass and heat transfer to spheres at Reynolds numbers sufficiently large for a velocity boundary layer to exist have received a considerable amount of attention in the literature. Experimental data have been obtained over wide ranges of N_{Re} and N_{Sc}, and a satisfactory theory has been developed for calculating the transfer rate at least up to the point of separation of the velocity boundary layer.

Mass transfer at Reynolds numbers below about 1, the range of Stokes flow, has received less attention This range is of importance for drops smaller than about 100 μ falling through air, bubbles smaller than about 100 μ rising through water, and for particles and drops moving through liquids. Transport in the Stokes flow region is relatively difficult to study experimentally, and only a few data are reported in the literature. Theory indicates that unlike the result for high N_{Re}, the Sherwood number (the Nusselt number for mass transport) is a function only of the Péclet number N_{Pe} and not of N_{Re} and N_{Sc} separately.

An approximate theoretical curve for the Sherwood number over the entire Peclet number range has been obtained by the author of this note (*1*) who assumed the existence of a concentration boundary layer and used the von Karman integral method. For $N_{Pe} = 0.3$, 1, 3, and 10 Yuge (*6*) has calculated what are presumably more accurate values for the Sherwood number by a numerical solution of the complete diffusion equation. A better curve than that given in (*1*) can be obtained by the use of the Yuge numerical data for low N_{Pe} and the Levich-Lighthill (*3*, *4*) thin boundary-layer method for $N_{Pe} \to \infty$.

For thin concentration boundary layers it is necessary to take into account only the first term in an expansion of the velocity in the region near the wall when the equation of convective diffusion is solved. For the region close to the surface of the sphere the diffusion equation can be written (*2*) as

$$u_1 \frac{\partial c_1}{\partial x_1} + v_1 \frac{\partial c_1}{\partial y_1} = \frac{2}{N_{Pe}} \frac{\partial^2 c_1}{\partial y_1^2} \quad (1)$$

The first terms in the expansion of the Stokes velocities give for u_1 and v_1

$$u_1 = (3/2)\, y_1 \sin x_1 \qquad (2)$$
$$v_1 = -(3/2)\, y_1^2 \cos x_1$$

Let $\eta = \alpha^{-1/3} y_1 \sin x_1 / (\int_0^{x_1} \sin^2 x'_1\, dx'_1)$ and assume $c = f(\eta)$. Substituting in (1) one gets

$$\frac{1}{N_{Pe}} \frac{d^2 c_1}{d\eta^2} + \frac{\alpha}{4} \eta^2 \frac{dc_1}{d\eta} = 0 \quad (3)$$

The solution which satisfies the boundary condition $c_1 = 1$ at $\eta = 0$ and $c_1 = 0$ at $\eta = \infty$ is

$$c_1 = 1 - \frac{\int_0^{\eta} e^{-\alpha N_{pe} \eta'^3/12} dn'}{\int_0^{\infty} e^{-\alpha N_{pe} \eta'^3/12} dn'} \quad (4)$$

The Sherwood number is given by

$$N_{Sh} = -\int_0^{\pi} \left(\frac{\partial c_1}{\partial y_1}\right)_{y_1=0} \sin x_1\, dx_1$$
$$= -\alpha^{-1/3} \left(\frac{dc_1}{d\eta}\right)_{\eta=0} \int_0^{\pi} \frac{\sin^2 x'_1\, dx'_1}{(\int_0^{x_1'} \sin^2 x_1''\, dx_1'')} \quad (5)$$

From Equation (4)

$$\left(\frac{dc_1}{d\eta}\right)_{\eta=0} = -\frac{(3/2)^{2/3}}{\Gamma(1/3)} (\alpha N_{Pe})^{1/3} \quad (6)$$

Hence (5) becomes*

* Using the thin concentration boundary-layer theory Natanson (5) finds for flow normal to a single cylinder:

$$N_{SH} = 1.17\, N_{Pe}^{1/3}/2(2 - \ln N_{RE})$$